

# Tax normalizations, the marginal cost of funds, and optimal environmental taxes

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## Abstract

Prior results on second-best optimal environmental taxes fail to hold under alternative tax normalizations, partly because the marginal cost of funds varies with the tax normalization. However, under any normalization, the optimal environmental tax typically lies below marginal damages. © 2001 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The issue of how to set environmental taxes in an economy with pre-existing distortionary taxes has attracted a great deal of attention. The literature on this issue has demonstrated that in such a second-best setting, the optimal environmental tax will be lower than marginal environmental damages (the optimal level in a first-best setting), and that the degree to which it is lower depends on the size of the pre-existing distortions.<sup>1</sup>

However, much of this literature employs a normalization of the tax system in which clean (non-polluting) goods are untaxed, while labor income and dirty (polluting) goods are taxed. Jaeger (1999) shows that the optimal environmental tax differential (the difference in tax rates between dirty and clean goods) varies with the tax normalization. He concluded from this that the prior literature's result that this differential will be less than marginal environmental damages (MED) resulted from the tax normalization used. Given the importance of this question — this literature has had an

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<sup>1</sup>See, for example, Bovenberg and de Mooij (1994); Goulder (1995); Parry (1995).

increasingly substantial influence on environmental policy — it would be troubling if the prior results were merely artifacts of the tax normalization.<sup>2</sup>

This paper investigates the normalization issue, employing an analytical general-equilibrium model that can consider an infinite range of possible tax normalizations. It confirms that the optimal environmental tax differential varies with the tax normalization. In addition, it shows that the prior literature's<sup>3</sup> result that the optimal tax differential equals MED divided by the marginal cost of public funds (MCPF) holds only under one possible tax normalization.

However, it also demonstrates that Jaeger's conclusion is incorrect. The ratio of the optimal tax differential to MED is constant across all possible tax normalizations, and will be less than one as long as there are pre-existing distortionary taxes and labor supply is not backward-bending.

In doing so, the paper shows that the MCPF varies with alternative tax normalizations. This last point has not been previously recognized by the literature on either the MCPF or second-best optimal environmental taxes, and is potentially important not just for environmental taxation, but for a broad range of other tax issues as well. It implies that researchers must be very careful not to use values for MCPF estimated under one tax normalization to reach conclusions about optimal taxation or public good supply in a context with a different tax normalization. It also indicates that comparisons of the MCPF between different tax systems (for example, comparing a consumption tax and an income tax) may prove to be highly misleading, due to the difference in normalization.

## 2. The model

A representative agent model is assumed, where households divide their time endowment ( $T$ ) between leisure ( $l$ ) and labor ( $L$ ), which is used to produce a dirty consumption good ( $D$ ),  $N$  clean goods ( $C_1 \dots C_N$ ), and a government-provided public good ( $G$ )<sup>4</sup>. The household utility function is

$$U(l, C, D, G) - \phi(D) \quad (1)$$

where  $U$  is continuous and quasi-concave. Each of the three goods is produced from labor under constant returns to scale, with units normalized such that one unit of labor can produce one unit of any of the goods.<sup>5</sup> Thus, production and the household time constraint are given by

$$T = l + L = l + \sum_i C_i + D + G \quad (2)$$

The government levies taxes on labor income (at the rate  $\tau_L$ ), consumption of the clean goods (at the rate  $\tau_C$ ), and consumption of the dirty good (at the rate  $\tau_D$ ). The unit normalization and assumption

<sup>2</sup>Fullerton (1997) and Schöb (1997) also considered the normalization issue, though with a different purpose than Jaeger (1999) or this paper; they demonstrated what was wrong with the conventional intuition that a higher government revenue requirement should imply heavier taxation of pollution. These two papers pointed out that under a normalization with no labor tax, the dirty-good tax rises with the government revenue requirement, but that this does not imply that the environmental tax differential rises, because the clean-good tax also rises.

<sup>3</sup>See, for example, Bovenberg and van der Ploeg (1994) or Parry (1995).

<sup>4</sup>This specification could also represent unproductive government spending (in which case).

<sup>5</sup>Note that this unit normalization is distinct from the tax normalization.

of competitive production implies that the pre-tax wage and pre-tax prices of the consumption goods will be equal, and we will normalize them to be 1. The consumer budget constraint is then

$$(1 + \tau_C) \sum_i C_i + (1 + \tau_D)D = (1 - \tau_L)L \quad (3)$$

This system of taxes is redundant in the sense that there are an infinite number of combinations of rates for these three taxes that would yield an equivalent consumer budget constraint. Multiplying the consumer budget constraint by any non-zero constant  $\alpha$  shows that the tax rates  $(\tau'_C, \tau'_D, \tau'_L)$  represent one such alternative normalization of the tax rates  $(\tau_C, \tau_D, \tau_L)$  where

$$\tau'_C = \alpha(1 + \tau_C) - 1, \quad \tau'_D = \alpha(1 + \tau_D) - 1, \quad \tau'_L = 1 - \alpha(1 - \tau_L) \quad (4)$$

This trivially shows that the optimal dirty-good tax varies with the tax normalization. In addition, it is straightforward to show that the optimal environmental tax differential  $(\tau_D - \tau_C)$  also varies. If the tax rates  $(\tau_C, \tau_D, \tau_L)$  are optimal, then under an alternative normalization (which will also be optimal), the environmental tax differential will follow.<sup>6</sup>

$$\tau'_D - \tau'_C = \alpha(\tau_D - \tau_C) \quad (5)$$

Tax revenue is used to finance a fixed level of the public good.<sup>7</sup> Thus, the government budget constraint is

$$\tau_C \sum_i C_i + \tau_D D + \tau_L L = G \quad (6)$$

The labor tax rate is assumed to vary to hold the level of  $G$  fixed for changes in the dirty-good tax. The result would be the same if the clean-good tax rate were varied instead.

Households maximize utility (1) subject to their time constraint (2) and budget constraint (3), taking prices, tax rates, the level of the public good, and the level of pollution as given. This yields the first-order conditions

$$U_C = (1 + \tau_C)\lambda; \quad U_D = (1 + \tau_D)\lambda; \quad U_L = (1 - \tau_L)\lambda \quad (7)$$

where  $\lambda$  is the marginal utility of after-tax income. Together with the constraints given previously, these implicitly define the (uncompensated) demand functions

$$C_i(\tau_C, \tau_D, \tau_L); \quad D(\tau_C, \tau_D, \tau_L); \quad l(\tau_C, \tau_D, \tau_L) \quad (8)$$

Taking a total derivative of utility (1) with respect to  $\tau_D$ , substituting in the first-order conditions (7), subtracting  $(1 + \tau_C)$  times the total derivative of the household time constraint (2) with respect to  $\tau_D$ , and rearranging terms yield

<sup>6</sup>Jaeger's (1999) result that the optimal environmental tax differential varies with the tax normalization represents a special case of this result, as it considers only one possible renormalization.

<sup>7</sup>The model would yield the same result under the alternative assumption that government revenue is spent on a lump-sum transfer to households, as long as the purchasing power of that transfer is held constant.

$$\frac{1}{\lambda} \frac{dU}{d\tau_D} = \left( \tau_D - \tau_C - \frac{1}{\lambda} \frac{\partial \phi}{\partial D} \right) \frac{dD}{d\tau_D} - (\tau_L + \tau_C) \frac{dl}{d\tau_D} \quad (9)$$

As Parry (1995) showed, the cost of an environmental tax depends on the degree of complementarity between the polluting good and leisure. If the polluting good is a relative complement to leisure, then the cost of taxing it will be lower than if it is a relative substitute. However, if both consumption goods are equal net substitutes for leisure, then the second term can be shown to equal<sup>8</sup>

$$(\tau_L + \tau_C) \frac{dl}{d\tau_D} = \left( \frac{\eta}{1 + \tau_C} - 1 \right) (\tau_D - \tau_C) \frac{dD}{d\tau_D} \quad (10)$$

where  $\eta$  is the marginal cost of public funds (MCPF), given by

$$\eta = \frac{(1 - \tau_L)(1 + \tau_C)}{1 - \tau_L - (\tau_L + \tau_C)\varepsilon_L} \quad (11)$$

This is the marginal cost to households of raising government revenue through a change in the labor tax or clean-good tax; thus, it is the ratio of the loss to households from a marginal increase in either of these taxes to the marginal revenue raised.<sup>9</sup>  $\varepsilon_L$  is the (uncompensated) labor supply elasticity, given by

$$\varepsilon_L = \frac{\partial l}{\partial \tau_L} \frac{1 - \tau_L}{L} \quad (12)$$

Substituting (10) into (9) and solving for the optimal environmental tax (tax differential between the clean good and the dirty good) yield

$$\tau_D - \tau_C = \frac{1 + \tau_C}{\eta} \frac{1}{\lambda} \frac{\partial \phi}{\partial D} \quad (13)$$

Marginal environmental damages (MED) is the marginal loss of utility from a unit of pollution, in money units. Under the tax normalization  $\tau_C = 0$ , expression (13) replicates the prior literature's finding that the optimal environmental tax is equal to MED divided by the MCPF. Under alternative tax normalizations, the prior literature's finding does not hold; in this general case, the ratio of the optimal environmental tax differential to MED is not  $1/\eta$ , but rather  $1 + \tau_C/\eta$ .

One might interpret this result as suggesting that the ratio of the optimal environmental tax differential to MED will vary across alternative tax normalizations. In fact, this is not the case. Substituting in expression (4), rearranging, and canceling out  $\alpha$ , one can show that under the alternative normalization  $(\tau'_C, \tau'_D, \tau'_L)$  — which follows expression (4) — this ratio equals

<sup>8</sup>This derivation is contained in a separate mathematical appendix, available from the author.

<sup>9</sup>This is a 'non-environmental' definition of the MCPF; it omits the effects of both the environmental tax differential (calculating both the cost to households and the revenue change as if the dirty-good tax were equal to the clean-good tax) and any change in environmental quality.

$$\frac{1 + \tau'_C}{\eta} = \frac{(1 - \tau'_L) - (\tau'_L + \tau'_C)\varepsilon_L}{(1 - \tau'_L)} = \frac{(1 - \tau_L) - (\tau_L + \tau_C)\varepsilon_L}{(1 - \tau_L)} \quad (14)$$

Thus, this ratio will remain constant across alternative tax normalizations. That is,  $\eta$  changes with  $\tau_C$  such that the ratio  $1 + \tau_C/\eta$  is constant. In addition, as long as the second term in the numerator is positive — which will be the case if labor supply is not backward-bending and the sum of the labor tax and clean good tax is positive — this ratio will be less than one. This indicates that the optimal tax differential will be less than MED, regardless of the tax normalization.

How is this result consistent with the result that the optimal tax differential varies with the tax normalization? The key insight is that changing the tax normalization will change the prices of the two consumer goods, and thus will change the marginal utility of income. As a result, MED — which is measured in money terms — will also change. Thus, the change in the optimal tax differential results not from a change in the ratio of that differential to MED — as Jaeger (1999) had concluded — but from a change in MED.

This result also might seem inconsistent with the earlier result from this paper that the ratio of the optimal tax differential to MED is not always equal to the MCPF. Again, there is no inconsistency; the MCPF varies under alternative tax normalizations. The reason for this is that the numerator of the MCPF — the marginal cost to households from increasing the tax rate — is implicitly measured relative to consumer prices, whereas the denominator — marginal tax revenue — is implicitly measured relative to producer prices. By changing the tax normalization, one alters the ratio of consumer to producer prices, and thus the MCPF changes as well. This result has not been noted previously in the literature either on the MCPF or on environmental taxation. It implies that researchers must be careful not to compare estimates of the MCPF taken from tax systems with different normalizations.

Thus, this paper has shown that while some of the prior results on optimal environmental taxes are sensitive to tax normalizations, the essential policy implication — that the optimal environmental tax differential will be lower in the presence of distortionary taxation — holds regardless of the normalization used.

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